1. Carefully state the floating point representation theorem. (5)

2. Carefully state the fundamental axiom of floating point arithmetic. (5)

3. Let \( a = 0.00123601 \) and \( b = 12360.1 \). Using 4 decimal digit rounding arithmetic, compute the following:
   (a) \( \text{fl}(a) \)
   (b) \( \text{fl}(b) \)

4. Consider the computation of \( c = ab \), where \( a \) and \( b \) are real numbers (not necessarily floats). Let the computed product be \( \tilde{c} \), and assume that none of \( a \), \( b \) or \( c \) overflow or underflow. Give a bound for the relative error in \( \tilde{c} \). (10)
5. On Conditioning and Stability

(a) Define ill-conditioning

(b) Describe what a condition number tells us.

(c) What is a backward stable computation?

(d) What is a backward stable method?

(e) Discuss how conditioning and stability can be used to evaluate the error in a computation.
6. Let $A = \begin{bmatrix} 1 & 9 \\ 6 & 3 \end{bmatrix}$. Let $\rho$ be the growth factor for Gaussian elimination.

(a) Give $L$, $U$, and $\rho$ from the $A = LU$ factorization of $A$.

(b) Give $L$, $U$, $P$, and $\rho$ from the $PA = LU$ factorization of $A$.

(c) Explain why pivoting is used in Gaussian Elimination and give an example of an invertible matrix that does not have an $LU$ factorization.
7. Let \( A \in \mathbb{R}^{n \times n} \) be nonsingular and \( b \in \mathbb{R}^n \). Describe how the \( PA = LU \) factorization can be used to solve the system \( Ax = b \). Give a flop count for each step.

8. Find the number of multiplications required to solve an \( n \times n \) lower triangular system using forward substitution.