1. Let \( f(x) = x^3 - 5 \). We’re looking for a zero of \( f \).

(a) Use the bisection method with \( a = 1 \) and \( b = 3 \) to find an interval of length strictly less than 1 which brackets a zero of \( f \).

(b) Use one iteration of Newton’s method to improve the guess \( x_0 = 1 \).

(c) Starting with \( x_0 = 1 \) and \( x_1 = 1.5 \), use one iteration of the secant method to find \( x_2 \).

(d) What is meant by order of convergence and what is the order of convergence for bisection, Newton’s and the secant methods?
2. State the floating point representation theorem (FRT).

3. Let \( a = 0.038734 \) and \( b = 42.089 \).

   (a) Compute the 3 (decimal) digit chopping representations of \( a \) and \( b \), call them \( \bar{a} \) and \( \bar{b} \) respectively.
   
   i. \( \bar{a} = \)
   
   ii. \( \bar{b} = \)

   (b) What is the unit roundoff, \( \mu \), for this arithmetic?

   (c) What do we mean by overflow and underflow?

   (d) What is digit cancellation?

   (e) Give a bound for the relative difference between 2 neighboring positive floats.
4. Let \( f(x) = x \sin(x) \).

(a) Compute \( P_2(x) \), the second Taylor polynomial for \( f \) at \( x_0 = 0 \).

(b) Use \( P_2 \) to approximate \( f(0.5) \).

5. Polynomial Roots

(a) Give a brief geometric description of Mueller’s method. (A well annotated picture would suffice.)

(b) Describe how deflation can be used to compute all of the roots of a polynomial.
6. Conditioning
   (a) What does *illconditioning* mean in a computational problem?

   (b) What is the absolute condition number for the problem
       “evaluate \( f(x) \) at \( x = x_0 \)?”?

   (c) Suppose \( f \) has only one zero. What is the absolute condition
       number for the problem “solve \( f(x) = 0 \)?”?

7. State the fundamental axiom of floating point arithmetic (FAFA).