1. Consider the DE $xy''' - 2y'' + (2/x)y' = 0$. (6)
   
   (a) Show that $1$, $x^2$, and $x^3$ form a fundamental set of solutions for this DE on $0 < x < \infty$.

   (b) Write down the general solution of this DE.

2. The DE $xy'' + y' = 0$ has a solution $y_1(x) = \ln(x)$. Find a second solution, $y_2$, so that $y_1$ and $y_2$ are linearly independent. (12)
3. In each of the following, write the solution $y$ in a form that doesn’t have complex numbers.

(a) Find the general solution: $y'' + 6y' + 9y = 0$

(b) Find the general solution: $y'' + 3y' - 4y = 0$

(c) Find the general solution: $y'' - 10y' + 29y = 0$

4. A certain spring satisfies Hooke’s Law.

(a) If a force of 5N stretches the spring 0.5m, what is the spring constant?

(b) A mass of 0.5 kg is attached. There is no driving force, but there is damping proportional to velocity with constant 10 Ns/m. Write the DE that models the motion of the mass, using $x(t)$ for the displacement.

(c) Write down the initial conditions corresponding to the following situation: The mass is pulled 0.2m below equilibrium and given an initial upward velocity of 3.0 m/s.
5. Explain whether the following system is underdamped, critically damped, or overdamped:

\[ 3x'' + 24x' + 48x = 0. \]

6. Find the general solution to \( y''' - 2y'' - 3y' = 2e^{-x} \). Hint:

\[ y_e = c_1 + c_2 e^{-x} + c_3 e^{3x}. \]
7. Solve \( x^3 y'' + 3x^2 y' - 8xy = 0. \) (10)

8. Find the general solution to \( y'' - (3/x)y' + (2/x^2)y = x^2. \) Hint: \( y_c = c_1 x + c_2 x^2. \) (12)