Show your work (imagine each answer is a short (math-language) essay).

1. Is the following system is underdamped, critically damped, or overdamped:

\[ 3x'' + 27x = 0. \]

2. Find the general solution to \( y'' = 12x - 6 \). Circle your answer.
3. Solve \( x^3y'' - 4x^2y' + 6xy = 0 \). Circle your answer.

4. Find the general solution to \( y'' - 4y' + 4y = (x + 1)e^{2x} \). Hint: \( y_c = c_1e^{2x} + c_2xe^{2x} \).
   You may leave your solution with integrals not evaluated. Circle your answer.
5. Let $L$ be a linear differential operator and suppose \{\(y_1, y_2\)\} is a fundamental set for $Ly = 0$ on an interval $I$.

(a) What can be said about $y_1$ and $y_2$?

(b) What order is the DE $Ly = 0$?

(c) Is $y_1 - y_2$ a solution to $Ly = 0$?

6. The DE $x^2 y'' + 2xy' - 6y = 0$ has a solution $y_1(x) = x^2$. Find a second solution, $y_2$, so that $y_1$ and $y_2$ are linearly independent. You may leave $y_2$ in an integral form. Circle your answer.

7. Find a linear differential annihilator for $x \cos(x)$. Circle your answer.
8. In each of the following, write the solution \( y \) in a form that doesn’t have complex numbers.

(a) Find the general solution: \( y'' - 2y' + 2y = 0 \). Circle your answer.

(b) Find the general solution: \( y'' + 4y' + 4y = 0 \). Circle your answer.

(c) Find the general solution: \( y'' + y' - 2y = 0 \). Circle your answer.

9. A certain spring satisfies Hooke’s Law.

(a) If a force of 5N stretches the spring 1m, what is the spring constant?

(b) A mass of 0.5 kg is attached. There is no driving force, but there is damping proportional to velocity with constant 2 Ns/m. Write the DE that models the motion of the mass, using \( x(t) \) for the displacement (not the solution: the DE).

(c) Write down the initial conditions corresponding to the following situation: The mass is pulled 0.5m below equilibrium and given an initial downward velocity of 2.0 m/s.