Show your work (imagine each answer is a short (math-language) essay).

(3) 1. Explain whether the following system is underdamped, critically damped, or overdamped:

   \[ 3x'' + 18x' + 27x = 0. \]

(12) 2. Find the general solution to \( y''' - 2y'' - 3y' = e^{3x} \). Hint: \( y_c = c_1 + c_2 e^{-x} + c_3 e^{3x} \).
3. Solve $x^3 y'' - x^2 y' - 8xy = 0.$

4. Find the general solution to $y'' - \left(\frac{3}{x}\right)y' + \left(\frac{2}{x^2}\right)y = x^2$. Hint: $y_c = c_1 x + c_2 x^2$. 


5. Consider the DE \( xy''' - 2y'' + (2/x)y' = 0 \).

(a) Show that 1, \( x^2 \), and \( x^3 \) form a fundamental set of solutions for this DE on \( 0 < x < \infty \).

(b) Write down the general solution of this DE.

6. The DE \( xy'' + y' = 0 \) has a solution \( y_1(x) = \ln(x) \). Find a second solution, \( y_2 \), so that \( y_1 \) and \( y_2 \) are linearly independent.
7. In each of the following, write the solution $y$ in a form that doesn’t have complex numbers.

(a) Find the general solution: $y'' - 10y' + 29y = 0$

(b) Find the general solution: $y'' + 6y' + 9y = 0$

(c) Find the general solution: $y'' + 3y' - 4y = 0$

8. A certain spring satisfies Hooke’s Law.

(a) If a force of 5N stretches the spring 0.5m, what is the spring constant?

(b) A mass of 0.5 kg is attached. There is no driving force, but there is damping proportional to velocity with constant 10 Ns/m. Write the DE that models the motion of the mass, using $x(t)$ for the displacement.

(c) Write down the initial conditions corresponding to the following situation: The mass is pulled 0.3m below equilibrium and given an initial upward velocity of 2.0 m/s.