General instructions:

(1) Do any four out of the five problems. If you choose to do five problems, we will not pick the best four out of your five solutions.

(2) Begin the solution to each problem by explaining, briefly, your approach to solving the problem. This will count for 5 points. The rest of the problem solving will count for 95 points.

(3) Write your I. D. code (not your name) and the problem number on each of your answer sheets.

(4) Use only one side of the paper for your answer. Furthermore, do not write your answers near the edge of the paper otherwise part(s) of your answer may not reproduce in photocopying.

(5) Hand in each problem as a unit. Number your pages. Turn in your formula sheet with your solutions.

Problems begin here.

Problem 1

(a) Two semi-infinite grounded conducting plates occupying $x = 0$, $y = 0$ planes intersect at right angle. A point charge $q$ is placed in the first quadrant at point $(x_0, y_0, z_0)$ [Fig. (a) below]. (30 pts)

(i) Find the potential and electric field in this region and show that your results satisfy the appropriate boundary conditions. Find the induced surface charge densities on the conductors at $x = 0$ and $y = 0$.

(ii) What is the force experienced by the point charge, if $x_0 = d = y_0$ and how much work was required to bring the charge $q$ from infinity to the point $x_0 = d = y_0$? This is the energy stored in the system. (15 pts)

(b) Using the results of part (a),

(i) Find the Green’s function for the Dirichlet boundary value problem in the first quadrant. (20 pts)

(ii) Now suppose that part of the conductor ($0 < x < a$) at $y = 0$ is maintained at constant potential $V_0$ and the rest of it and the conductor at $x = 0$ are kept grounded [Fig. (b)]. Use the Green’s function you found in (a) to write an expression for the scalar potential in the first quadrant ($0 < x, y < \infty$). Do not attempt to evaluate the integrals but show the integrand and all the limits of integration explicitly. (30 pts)
Problem 2
A long circular cylindrical magnet of radius $a$ has uniform permanent magnetization $M$ perpendicular to its axis. Find $\mathbf{B}$ and $\mathbf{H}$ fields both inside and outside the cylinder.

Problem 3
In the total internal reflection of light the electric field of the transmitted wave has the (complex) form

$$\mathbf{E}(x, z, t) = e_y e_0 e^{i(kr - \omega t)}$$

where $k = \frac{\omega}{c} (pe_x + iq e_z)$,

and $p, q$ are real numbers that depend on the angle of internal incidence $\theta$ and the refractive index $n$ of the medium of incidence.

(a) Find the physical (real) electric and magnetic fields. Is this a transverse wave? Is the electric field perpendicular to the direction of propagation? Is the magnetic field perpendicular to the direction of propagation? (40 pts)

(b) Compute the time averaged electric and magnetic energy densities and the Poynting vector for this wave. How are these quantities related? (45)

Problem 4
An insulating ring of radius $R$ lies in the $x-y$ plane, centered at the origin. It carries a linear charge density $\lambda = \lambda_o \sin \phi$, where $\lambda_o$ is a positive constant and $\phi$ is azimuthal angle. The ring is spinning at a constant angular velocity $\omega$ about the $z$-axis.

(a) Find the leading term in the vector potential for this system in the radiation zone. Which electric or magnetic multipole contributes to this? (40 pts)

(b) Find the average intensity of radiation at a point $(r, \theta, \phi)$ in the radiation zone. Find and sketch the time averaged angular distribution of radiated power. What is the total power radiated by this system? (45 pts)

Problem 5
An ideal magnetic dipole moment $\mathbf{m}$ is at rest at the origin of an inertial frame $K'$ that is moving with uniform speed $v_x = v$ in the $x$-direction with respect to another inertial frame $K$. In $K'$, the vector potential of the magnetic dipole at a field point with position vector $r'$ is given by

$$\mathbf{A}' = \frac{\mu_o}{4\pi} \frac{\mathbf{m} \times r'}{r'^3}$$

and the electric potential $\Phi' = 0$.

(a) Find the scalar potential in frame $K$ and express it in terms of the coordinates of the field point in the reference frame $K$. (45 pts)

(b) Show that in non-relativistic limit, the scalar potential in $K$ is that of an ideal electric dipole

$$\mathbf{p} = \frac{v \times \mathbf{m}}{c^2}$$

located at the origin of $K'$. (40 pts)