General instructions:

(1) Do any four out of the five problems. If you choose to do five problems, we will not pick the best four out of your five solutions.

(2) Begin the solution to each problem by explaining, briefly, your approach to solving the problem. This will count for 5 points. The rest of the problem solving will count for 95 points.

(3) Write your I. D. code (not your name) and the problem number on each of your answer sheets.

(4) Use only one side of the paper for your answer. Furthermore, do not write your answers near the edge of the paper otherwise part(s) of your answer may not reproduce in photocopying.

(5) Hand in each problem as a unit. Number your pages. Turn in your formula sheet with your solutions.

Problems begin here.

Problem 1
A charge $q$ of mass $m$ is in free space above a perfectly conducting ground plane. The charge is released at $t = 0$ from rest at $z = d$. Ignore the effect of gravity.

(a) Find the velocity of the charge as a function of $z$.

(b) How long before the charge hits the plane at $z = 0$.

Hint: You may find the following integral useful

$$\int \frac{\sqrt{z}}{\sqrt{d-z}} \, dz = -\sqrt{z(d-z)} + d \tan^{-1} \sqrt{\frac{z}{d-z}}.$$ 

Problem 2
Two halves of a solid metallic conductor of radius $a$ and infinite conductivity are separated by a very small insulating gap. An alternating potential is applied between the two halves of the sphere so that the potentials are $\pm V_0 \cos \omega t$.

(a) In the long wavelength limit $\omega a/c \ll 1$ and far from the sphere, $\omega r/c \gg 1$ (radiation zone), find the leading contributions to the electric and magnetic fields. One of these fields is perpendicular to the plane containing the $z-$axis and the radius vector to point $P$. Which one is it?

(b) Find the angular distribution of average power radiated by the sphere and identify the directions of maximum and minimum radiation.

(c) How much power is radiated by the sphere and how does it depend on $\omega$, $a$ and $V_0$. 

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Problem 3

Consider $B$ and $H$ fields in circular cylindrical coordinates $\rho$, $\varphi$, $z$:

$$B(\rho, \varphi, z) = \begin{cases} \mu_0 Ae_x, & \rho < R \\ \frac{\mu_0 AR^2}{\rho^2} (e_\rho \cos \varphi + e_\varphi \sin \varphi), & \rho > R \end{cases}$$

$$H(\rho, \varphi, z) = \begin{cases} -Ae_x, & \rho < R \\ \frac{AR^2}{\rho^2} (e_\rho \cos \varphi + e_\varphi \sin \varphi), & \rho > R \end{cases}$$

Here $e_x$ is a unit vector in the direction of $x-$ axis, and $e_\rho$ and $e_\varphi$ are unit radial and azimuthal vectors in cylindrical polar coordinates, and $A$ and $R$ are constants.

(a) Find magnetization $M$ (magnetic moment per unit volume) everywhere.

(b) Find all free and magnetization current densities, both volume and surface types.

Problem 4

A rectangular waveguide of sides $a$ and $b$ ($a > b$), infinitely long in the $z$ direction, is used for guided wave propagation in the TM mode. For TM modes the magnetic field is perpendicular to the direction of propagation $z$. The walls are perfect conductors.

(a) What are the differential equations and the boundary conditions satisfied by the $E$ and $B$ fields at the walls? For the TM mode, you can express all field components in terms of a single field component. Which field component is that? Write down the differential equation and the boundary conditions satisfied by it.

(b) Find an expression for the lowest frequency that can propagate in the waveguide. If $a = 4$ cm and $b = 3$ cm, can you propagate a wave of frequency $\omega = 6.0 \times 10^{10}$ rad/s?

(c) How far can the waves of frequency equal to half the cutoff frequency penetrate the waveguide? You can take the distance over which the power drops to its $1/e$ of its initial value to be a measure of the penetration depth.

Problem 5

A point electric dipole $p_o$ is located at the center of a spherical cavity inside an infinite medium of dielectric constant $\epsilon$. The radius of the cavity $a$. It is desired to find the electric potential both inside and outside the spherical region.

(a) Write down the boundary conditions that the potential must satisfy.

(b) Find the electric potential and field everywhere.

(c) Find the polarization charge density on the walls of the cavity and show that it produces a uniform field inside the cavity.