2013 Candidacy Examination

Part A: Classical Mechanics

January 8, 2013

General Instructions:

Do any four out of the five problems. Return solutions to only four problems.

Read each problem carefully before beginning.

Put the problem number on each page.

Put your I.D. code on the top of each page. This I.D. should not be your student I.D. number, simply select a number or code name which will identify you only to yourself. Then a sheet will be passed around so that each student can give their name and accompanying I.D. code and this sheet will be placed in a sealed envelope.

Write only on one side of each sheet of paper.

Number your pages.

Write clearly and neatly.

Hand in each problem as a unit. Turn in your formula sheet with your problem solutions. Make sure your name is NOT on the formula sheet, only your I.D.!

IMPORTANT: Explain your approach to solving the problem before you begin. This will count 5 points. The rest of the problem will count 95 points.
Problem 1: Sliding Ramp

A block of mass $m$ slides without friction on an inclined plane which is one surface of a ramp of mass $M$, whose horizontal surface is free to slide without friction on a horizontal plane. The angle between the inclined plane and the horizontal is $\alpha$. Draw a diagram of the system, labelling all forces on both the block and the plane.

a) Write sufficient equations to find the motion of the block and the ramp. You do not need to solve these equations (40 points).

b) Show that the normal force between the block and the ramp is given by

$$N = \frac{mg \cos \alpha}{1 + \frac{m}{M} \sin^2 \alpha}$$

(1)

(25 points).

c) Find the velocities of the block and the ramp when the block reaches the bottom of the ramp, if the block began at a vertical height $h$ above the plane upon which the ramp rests. Give these velocities in terms of $m$, $M$, $h$ and $\alpha$ (30 points).

Problem 2: Killer Stars

If a supernova goes off too close to the Earth most lifeforms would be destroyed by the gamma-ray flux. The kill radius $k$ is about 20 light years. The average spacing between stars in our part of the Galaxy is about 3 light years, but supernova progenitors are rare, about 1 in $10^6$ stars. The Sun moves through the stellar neighborhood at a speed of about 30 km/s. Supernova progenitors have a life span, from birth to destruction, of about $10^7$ years, compared to which the duration of the supernova itself is negligible. What is the mean time between deadly exposures to supernova radiation for the Earth?

A light year is about $9.5 \times 10^{12}$ km.
Problem 3: The Slingshot Tube

a) Show that the moment of inertia of a thin rod about its center of mass is $ml^2/12$ (15 points).

b) A long thin tube of negligible mass is pivoted so that it may rotate without friction in a horizontal plane. A thin rod of mass $M$ and length $l$ slides without friction in the tube. Choose a suitable set of coordinates and write Lagrange’s equations for this system (30 points).

c) Initially the rod is centered at the pivot and the tube is rotating with angular velocity $\omega_o$. Show that the rod is unstable in this position and describe, both quantitatively and qualitatively, its subsequent motion if it is disturbed slightly (30 points).

d) After a long time has passed what are the radial and angular velocities of the rod (20 points)?

You may find the following indefinite integral useful:

$$\int \frac{l^4 \omega_o^2 x}{(12x^2 + l^2)^2} dx = \frac{1}{12} \frac{l^2 \omega_o^2 x^2}{12x^2 + l^2} + C$$

(2)

where $C$ is a constant of integration

Problem 4: Atwood’s Machine

An Atwood’s machine is constructed from 5 massless pulleys connected by an extensionless string wound around each pulley. Two of the pulleys are suspended at fixed height from the ceiling, the other three pulleys are suspended only by the extensionless string, which is fixed to the ceiling at either end. From the three unfixed pulleys hang masses which, arranged from left to right, have values $4m$, $3m$ and $m$ (see the accompanying figure). Let $x$ be the height of the leftmost mass (with mass $4m$ suspended from it) and $y$ be the height of the rightmost mass (with mass $m$ suspended from it). Find the conserved momentum of this system.
Problem 5: Tidal Forces

Consider a spacecraft in orbit around a planet of mass $M$. Inside the spacecraft is a droplet of liquid, which in the reference frame of the spacecraft appears to be weightless and floating in midair. The center of the drop of liquid is a distance $R$ from the planet. Assume that at time $t = 0$ the droplet is spherical with radius $r$ and surface tension forces can be neglected.

a) Considering only nonrelativistic gravitational forces due to the planet, compute the $x$ and $y$ components of accelerations at time $t = 0$ of
i. the point A on the droplet closest to the planet
ii. the point B at the center of the droplet
iii. the point C at the side of the droplet

Express your answers in terms of $R$, $r$, $M$, and the gravitational constant $G$ (40 points).

b) Show that the droplet undergoes stretching along the line AB and compression along the line CB. Using the fact that $r \ll R$ and approximating to first order in $r/R$ show that stretching happens at twice the rate of compression. Continue to neglect surface tension (55 points).