General Instructions

Do any four out of the five problems. Return solutions to only four Problems.

Put your I.D. code on the top of each page. This I.D. code should not be your student I.D. number, simply select a number or code name which will identify you only to yourself. A sheet will be passed around so that each student can give their name and accompanying I.D. code and this sheet will be placed in a sealed envelope.

Read each problem carefully before beginning.

Put the problem number and your I.D. code at top center of each page.

Number each page after problem number and your I.D. code.

Write only on one side of each sheet of paper.

Write clearly and neatly.

Hand in each problem as a unit. Turn in your formula sheet with your problem solutions. Make sure your name is NOT on the formula sheet, only your I.D. code!

IMPORTANT: Explain your approach to solving the problem before you begin. This will count 5 points. The rest of the problem will count 95 points.
2015 Classical Mechanics Candidacy Examination
Do any 4 of the problems numbered 1 to 5 below

Problem 1.

(a) Fixed axis of rotation  (b) Frictionless plane

A ruler of length $L$ is held vertically and allowed to topple over in constant gravity $g$ onto a desktop. In case (a) (see above left) the ruler’s lower end pivots in a groove that acts as a fixed rotation axis. In case (b) (see above right) the ruler’s lower end slides frictionlessly along the desktop. For each case, derive the speed with which the upper end of the ruler hits the desk and compare each speed with the speed of single-particle free fall from a height equal to the length $L$ of the ruler.

Explanation of approach: 5pts    Case (a): 35pts       Case (b): 60pts

Problem 2.

One can model a disk-type galaxy as an infinitely thin horizontal collision-free mass plane of large radius with a small uniform spherical bulge in the center. The masses and sizes of individual stars are infinitesimal compared to these components. Assume mass of the bulge is $M$ and the surface density (in units like $kg/m^2$) of the disk is $\sigma$.

(a) 30pts  What is the orbital period $P$ of a star with a circular orbit of radius $R$ orbiting somewhere in the middle of the disk ?

(b) 50pts  If this star has a small maximum velocity $V_p$ perpendicular to the disk, what is its period of oscillation $T$?

(c) 15pts  Is its vertical motion simple harmonic motion ? Discuss why or why not.

Problem 3.

Two $M=1\text{kg}$ balls are connected by springs $k_1=16\text{N}\cdot\text{m}^{-1}$ and $k_2=37\text{N}\cdot\text{m}^{-1}$ to ends of a box and coupled to each other by spring $k_{12}=36\text{N}\cdot\text{m}^{-1}$. (See figure.) They slide on frictionless rod and come to rest at $x=0=y$.

(a) 20pts Find Lagrangian equations of $(x,y)$-motion. Write them in a coupling matrix ($K$-matrix) form.

(b) 35pts Find each eigenmode and eigenfrequency and sketch their direction of motion on an $x,y$-graph.

(c) 40pts If system starts at rest with $(x(0),y(0))=(1,0)$ derive its subsequent motion and sketch the resulting path in the $x,y$-plane. Show algebraically that their path is exactly a parabola.
Problem 4.

Consider a point mass \( m \) and a ring with mass density \( \rho \) (mass/length). Assume that the point mass is positioned on a line that is perpendicular to the plane of the ring and passes through the center of the ring (dashed line in diagram). Both the ring and the mass are free to move. Assume that no forces act except gravitational attraction between the ring and the point mass.

(a) 55 pts Determine the equations of motion for the ring and the point mass.

(b) 40 pts Show that, for sufficiently small \( z \), the motion of both bodies is simple harmonic oscillations. Determine the frequency of the oscillations.

Problem 5.

An astronaut (mass \( m \)) on a spacewalk finds that his jet-pack has failed and that his only connection to the spaceship (mass \( M \)) is by a wire of length \( L \). (See figure above.)

(a) 40 pts Assuming that the spaceship and the astronaut line up along a radial direction from the center of the earth, derive formula for the tension in the wire.

(b) 55 pts Assume the height of the spaceship is negligible compared to the Earth's radius \((R=6400 \text{ km})\). Approximate acceleration due to gravity by that of Earth's surface \( g \sim 10 \text{m/s}^2 \). Let astronaut including the space-suit be mass \( m=100 \text{ kg} \) and negligible compared to the mass \( M \) of the spaceship. Let tethering wire length be \( L=100 \text{m} \). From all this, estimate numerical value of wire tension.