CANDIDACY EXAMINATION : ELECTROMAGNETISM
TIME : 9:00 am - 1:00 PM, Wednesday, January 7, 2015

General instructions:

(1) Do any four out of the five problems. If you choose to do five problems, we will not pick the best four out of your five solutions.

(2) For each problem, begin by explaining, briefly, your approach before solving the problem. This will count for 5 points. The rest of the problem solving will count for 95 points.

(3) Write your I. D. code (not your name) and the problem number on each of your answer sheet.

(4) Write only on one side of the answer paper. Furthermore, do not write your answers near the edge of the paper otherwise part(s) of your answer may not reproduce in photocopying.

(5) Hand in each problem as a unit. Number your pages. Turn in your formula sheet with your solutions.
Problems begin here.

Problem 1:

(a) A long thin straight wire of length $L$ and radius $R$ lies parallel to, and a distance $d$ from, a large plane metal sheet which is connected to ground [see Fig. 1(a) below]. The wire carries a charge $Q$ which is uniformly distributed. Assume $R \ll d \ll L$. Find the direction and magnitude of the force per unit length on the wire. Ignore end effects and gravity.

(b) A long thin wire of length $L$ and mass $m$ is stretched under tension $T$ between two supports. The wire now lies halfway between two large parallel plane metal sheets, both of which are connected to ground [see Fig. 1(b) below]. The charge $Q$ on the wire is adjustable. The distance from the undisplaced wire to each sheet is $d$. If a segment of the wire near the midpoint is given a small displacement toward one plate and then released, what is the direction and magnitude of the instantaneous net force on this segment?

(c) Describe the motion of the wire after release. What happens as $Q$ is increased from zero to a larger and larger value?

\[ \text{Fig 1(a) } \quad \text{Fig 1(b)} \]

Problem 2:

(a) A sphere with radius $R$ rotates with constant angular velocity $\omega$. A uniform charge distribution is fixed on the surface. The total charge is $q$. Calculate the magnetic moment $\mu$.

(b) Assume that the mass distribution is uniform throughout the sphere. Express magnetic moment of part (a) as a factor times the mechanical angular momentum $\vec{L}$. Compare the thus obtained expression with that of an electron,

\[ \vec{\mu} = g \frac{q}{2m_e c} \vec{s}; \quad g \approx 2, \]

where $m_e$ and $\vec{s}$ are mass and spin of the electron respectively.

Problem 3: A plane electromagnetic wave with electric field

\[ \vec{E}(\vec{r}, t) = \Re \left[ \left( \frac{\vec{x} + i \vec{y}}{\sqrt{2}} \right) A e^{(i\omega t - \vec{k} \cdot \vec{r})} \right] \]

is propagating in vacuum.
(a) Determine the wavelength, frequency, phase velocity, and polarization of the incident wave. Find the magnetic field of the incident wave and express it in terms of given quantities.

(b) What are the Poynting vector, momentum density, and energy density of the incident wave? How these three quantities are related?

(c) The plane wave described in part (a) passes through a linear polarizer with its axis parallel to the $x$-axis. What is the electric field of the wave after passing through the polarizer? This wave is normally incident on a conductor in the region $z \geq 0$. Assume that the conducting medium is highly absorbing with permeability $\varepsilon \approx i\epsilon_0\alpha^2$ and $\mu \approx \mu_0$, where $\alpha \gg 1$. What is the wavelength and skin depth in the conducting medium? Write down the boundary conditions at the conductor surface. Using these boundary conditions derive coefficient of reflection to the first order in $1/\alpha$.

Problem 4: An alternating current $I = I_0 \cos(\omega t)$ flows down a long straight wire, and returns along a coaxial conducting tube of radius $a$ [see Fig. 4].

(a) In what direction does the induced electric field point (radial, circumferential, or longitudinal)?

(b) Assuming that the field goes to zero as $r$ goes to infinite, find $\vec{E}(r, t)$, where $r$ is in radial direction.

(c) Find the displacement current density $\vec{J}_d$

(d) Integrate it to get the total displacement current,

$$I_d = \int \int \vec{J}_d \cdot dA$$

(e) Compare $I_d$ and $I$, what is their ratio? If the outer cylinder is, say 2 mm in diameter, how high would the frequency have to be, for $I_d$ to be 1% of $I$?

![Fig 4]

Problem 5: In a reference frame a static, uniform, magnetic induction $B_0$ is parallel to the $y$ axis, and a static, uniform electric field $E_0 = CB_0/3$, lies in x-y plane, making an angle $\theta$ with the $y$ axis. Determine the relative velocity of a reference frame in which the electric and magnetic fields are parallel. Determine velocity of the frame in the limit $\theta \ll 1$ and find both the fields in this reference frame. If $\theta \neq \pi/2$ can the electric field be perpendicular the magnetic field in a reference frame?

[Hint: transformation for the $E$ and $B$ fields are]

$$\vec{E}' = \gamma \left[ \vec{E} + c(\vec{\beta} \times \vec{B}) \right] - \frac{\gamma^2}{1 + \gamma} \vec{\beta} (\vec{\beta} \cdot \vec{E}), \quad \vec{B}' = \gamma \left[ \vec{B} - (\vec{\beta} \times \vec{E})/c \right] - \frac{\gamma^2}{1 + \gamma} \vec{\beta} (\vec{\beta} \cdot \vec{B}).$$

Here $\vec{\beta} = v/c$ and $\gamma = 1/\sqrt{(1 - \beta^2)}$.]