A1.
Three rigid spheres are connected by light, elastic rods, as shown in the figure. The relative masses are \( m_1, m_2, m_3 \neq 1:2:1 \).

Assuming small oscillations, so the longitudinal oscillations do not couple to the transverse oscillations, describe all the normal modes of the system, sketch them, and state all you can about their relative frequencies. Ignore translations of the center of mass and rotations of the molecule as a whole, that is, restrict yourself to motions with (total) \( P=E=0 \).

A2.
A pendulum of mass \( m \) is suspended from the cusp of a cycloid cut in rigid support (see figure). The path described by the pendulum bob is a cycloid, given by the equations

\[
x = a(\theta - \sin \theta)
\]

\[
y = a(\cos \theta - 1)
\]

where the length of the pendulum is \( b=2a \), and \( \theta \) is the angle of rotation of the circle generating the cycloid. Find the Lagrangian, the Lagrange equations of motion, and the time period of the oscillations.

A3.
A mass \( m \) moves in a circular orbit of radius \( r_0 \) under the influence of a central force whose potential is \(-kr_0^2\).

(a) If the angular momentum of the particle is \( L \), write down an expression for the effective radial potential seen by the particle, as a function of \( r \).

(b) Use the result in (a) to find an expression for the value of \( r \) leading to a circular orbit, for the given \( L \).

(c) Show that this circular orbit will be stable under small oscillations (that is, the mass will oscillate about the circular orbit) if \( n < 2 \).
B1. A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart. A resistor $R$ is connected across the rails and a uniform magnetic field $B$ pointing into the page, fills the entire region.

a.) If the bar moves to the right at speed $v$, what is the current in the resistor? In what direction does it flow?

b.) What is the magnetic force on the bar? In what direction?

c.) If the bar starts out with speed $v_0$ at time $t=0$, and is left to slide, what is its speed at a later time $t$?

d.) The bar eventually stops moving. Where does its kinetic energy go? Prove that energy is conserved (that this other form of energy acquire all of the initial kinetic energy).

B2. An electric dipole of moment $p$ is placed at a distance $d$ from a grounded conducting plane. The dipole makes an angle of 90° with the plane.

(a) Find the force experienced by the dipole.

(b) How much work is required to move the dipole to infinity?

B3. Consider a wire of length $2L$ and radius $a$ carrying a current $I$ that is uniformly distributed over the cross section of the wire. Find an expression for the external and internal vector potential $\mathbf{A}$ and magnetic field $\mathbf{B}$ at a point equidistant from the ends of the wire (assume $L > a$).
A. Consider a gas of mesoscopic mole–mole particles in perpetual volume $\frac{dV}{f}$

$$\left( \frac{fV}{g} \right) = \left( \frac{fV}{g} \right)$$

B. Consider the expression for the density for which $(s)$ is a mediator.

$$\left( \frac{fV}{g} \right)$$

C. Consider the constraint on the density, then for the case $B$

where $(s)$ is the constraint.

$$\frac{dV}{f}$$

D. Consider the constraint on the density, then for the case $B$

where $(s)$ is the constraint.

$$\frac{dV}{f}$$

E. According to the knowledge of mixed gas pressure, we get the total of the collisions.
Using the vector model, find the expression for the angle $\theta$ between the total angular momentum $J$ and the total magnetic moment $\mu$. Your answer should be in terms of the orbital and intrinsic magnetic moments. Non-consider the specific case of the $3p$ state of Na. The two line structure of the $3p \to 3s$ transition have wave numbers $\lambda_i = 5656$ and $\lambda_s = 5890$ Å. Using this information, determine the transition frequency of Na in the $3p$ state of Na.

The energy level diagram for the rotational levels in each of the two lowest-lying electronic states of the sodium ground state is shown. The $\nu$ labels the vibrational level.

(b) Find the energy of the transitions that give rise to the two observed peaks in the infrared spectrum. Assume that the molecules are identical and that any direct rotational or vibrational-rotational transition would be observed. What is the origin of $\nu = 0$? What is the origin of $\nu = 0$? What is the origin of $\nu = 0$? What is the origin of $\nu = 0$?

(c) Calculate the zero-point vibrational energy.

(d) In a molecular reaction $H_2 + \text{Be} \rightarrow \text{BeH}_2$, alpha particles from $\gamma$ rays, with energy of $5.40 \text{ MeV}$ bombard Be to target at rest.

(e) Find the $Q_i$ value of the reaction.

(f) Find the energy of the Be atoms at angles $\phi = 90^\circ$, $180^\circ$, $270^\circ$, and $360^\circ$.

(g) Calculate the energy levels of Be in each vibrational level.
D4. The concentration of charge carriers in a semiconductor is a critical parameter in device performance. Using the definition of current as being the rate of charge flowing past a surface \( I = \frac{\Delta Q}{\Delta t} \),

(a) Find an expression for the number of charge carriers per unit volume \( n \), in terms of the amount of current flowing through the semiconductor, \( I \), the fundamental unit of charge \( q \), the average speed of the charge carrier, \( v \), and the rectangular area of the surface the charge flows through, \( A = w 	imes h \).

(b) Why is it impossible to use this formula to measure the concentration of carriers?

(c) If a magnetic field is applied perpendicular to the current flow, draw a diagram showing the forces that act on the charge carrier in a direction perpendicular to both the current flow and the magnetic field.

(d) Explain in words what happens to a negatively charged particle entering the region with the magnetic field. Repeat for a positive charge.

(e) Balance the forces found in part (c) to predict the voltage generated, \( V \). Combine this result with your result from part (a), to find a new expression for \( n \). Explain why this formula makes it possible to determine \( n \).
E1. Imagine that the electron in a hydrogen atom is in the combined position-spin state

\[ R \left( \frac{3}{4} \right) \begin{pmatrix} Y^+ \psi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} Y^+ \psi \rightarrow \frac{1}{\sqrt{2}} \end{pmatrix} \end{pmatrix} \]

Here \( R \) is the radial wavefunction \( R(1) = \frac{1}{\sqrt{2}} a^{1/2} \frac{e^{-r/a}}{k} \exp \left( \frac{-r}{2\alpha} \right) \), \( Y^+ \) is the spherical harmonic \[ Y^\pm \left( \frac{3}{4}, \frac{3}{4} \right) \cdot \right. \]

\[ = \frac{3}{4\pi} \left( \begin{array}{cc} \cos \theta \sin^2 \phi \end{array} \right), \]

and \( \psi \) and \( \psi' \) are "spin up" and "spin down" states (relative to the x-axis), sometimes written as \( \chi^+ \) and \( \chi^- \) instead.

(a) If you measured the orbital angular momentum squared \( (L^2) \), what values might you get, and what is the probability of each?
(b) Same for the z-component of orbital angular momentum \( (L_z) \)
(c) Same for the spin angular momentum squared \( (S^2) \)
(d) Same for the z-component of spin angular momentum \( (S_z) \)

Let \( \mathbf{J} = \mathbf{L} + \mathbf{S} \) be the total angular momentum.

(e) If you measured \( \mathbf{J} \), what values might you get, and what is the probability of each?
(f) Same for \( J_z \)
(g) If you measured the position of the particle, what is the probability density for finding it at \( r, \theta, \phi \)?
(h) If you measured both the z-component of the spin and the distance from the origin (note that these are compatible observables), what is the probability density for finding the particle with spin up and at a radius \( r \)?

E2.

Consider a particle of mass \( m \) in a one-dimensional box of length \( L \), that is, let it be in an infinite square-well potential as indicated in the Figure.

(a) Calculate the energy of the two lowest stationary states according to standard quantum mechanics. Give energy values and normalized wavefunctions.
(b) Suppose the particle has probability $P_f$ to be in the first or lowest state and probability $P_2$ to be in the second or next lowest level where $1 - P_f - P_2$. Calculate the expected energy of the particle for a 50-50 combination ($P_f = P_2$).

(c) Calculate the expected position $(x, y, z)$ as a function of constants $m$, $q$, $E$, $h$, and probabilities $P_f$ and $P_2$.

(d) Use your result from (c) to tell what $P_f$ and $P_2$ give the maximum $(x)$ and what that value is as a fraction of length $L$. (Numerical answer only will be accepted for this part.)

E.3. Calculate the probability current density $j = -i(h/2\mu)\left[\psi^* \nabla \psi - \psi \nabla \psi^*\right]$ for the hydrogenic wavefunction $\psi(r, \theta) = R_n(r) \Theta_n(\theta) e^{-\mu r^2}/r$, where $\mu$ is the reduced mass, and $R_n$ and $\Theta_n$ are real functions of $r$ and $\theta$, respectively. Interpret your result.

Note: in spherical coordinates the gradient operator is

$$\nabla = \hat{\xi} \frac{\partial}{\partial r} + \hat{\eta} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{\zeta} \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi}.$$